The Quaternion

The Newsletter of the Department of Mathematics, USF–Tampa

Volume 17: Number 1 Fall, 2002

Leopard Spots, Emergent Properties, and Ghostly Mathematics

Richard Stark is a Professor of Mathematics at USF. Professor Stark was Department Chair from 1994 to 1998.

Many scientists are now studying emergent phenomena. We see these phenomena when a complex system (like a termite colony) without any bluprints or formal lines of authority somehow generates an orderly, or even beautiful, overall structure (like a termite mound, with natural air conditioning). We see them in morphogenesis – when a single cell evolves into a complex living creature, such as the reader.

I cannot define "emergence" as used in mathematics, but I have interesting emergent phenomena in mathematical biology. I will start with some mathematical examples.

The fact that 2 + 2 equals 4 is true, but it is not an emergent phenomenon. For one thing, "2 + 2 = 4" is just an assertion about two numbers, not about the structure of the real line. For another, it is verifiable by straightforward algebraic manipulations, while emergent phenomena often require some higher-order argument. Emergent truths tend to be unexpected.

On the other hand, to prove that the cardinality of the reals is greater than that of the integers, one uses a "diagonalization" argument (any one-to-one mapping of the integers into the reals must miss a real). This proof involves individual numbers below and the entire number line above, in a ghostly interplay.

A similar same agrument proves that most real numbers are "undefinable." By the nature of our languages, the definitions of real numbers may be listed and paired with the integers. Thus the set of definable reals is countable and thus has length 0. But the set of all reals in $[0, 1]$ is of length 1, so

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Departmental News

Two external committees came to evaluate our department, and our Graduate and Undergraduate programs received high praise from the reviewers. The representatives from Pennsylvania State, Georgia State, and Cornell gave us outstanding marks. "Like an atom: small but powerful," was the summation by Dr. Hubbard of Cornell. (We are exploding with pride).

Led by our new Office Manager, Frances Johnston, the math office run by Beverly Hoffmeyer, Maureen Kearse, Denise Marks, Mary Ann Wengyn and the rest of the staff has won unanimous praise from the students and faculty. Thank you for making our life enjoyable!

The department had four visitors from Taiwan, Bulgaria and Denmark.

Due to the aggressive recruitment policies and personal efforts of our Graduate Program Coordinator, Jim Tremmel, and the Gruduate Admissions Director, Natasha Jonoska, the number of graduate students in the department has doubled.

Severe budget cuts continue to plague our programs: most Summer courses were cancelled because of the budget crisis. (If you see any of us begging on the streets: Help! Give generously!)

Departmental People

Ralph Oberste-Vorth is leaving USF to become Head of the Division of Mathematics at Marshall University in Huntington, West Virginia. Professor Oberste-Vorth received his Ph.D. from Cornell University in 1987, where he worked on topology over complex domains under John Hubbard. He came to USF in 1989, was tenured in 1995, and became Associate Chair in 1998. He oversaw the undergraduate program, and continued to do research in topology, especially in complex dynamics. And just a few months ago, he and his wife Jamie had a new baby boy named Troy. We wish them well in their future adventures.

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almost all reals must undefinable.

One can see the other side of the coin in a typical undergraduate course in linear algebra. The proofs and concepts are built as concretely and predictably as the bricks in Wrigley Field's wall. I cannot imagine an emergent truth being found here.

On the other hand ... In his beautiful little book "A Primer of Real Functions," Ralph Boas used the Baire Category Theorem to prove that almost every continuous real function is nowhere differentiable. And, L. E. J. Bouwer proved that every continuous function f mapping a the unit interval $[0, 1]$ into itself has a fixed-point (i.e., a point x such that $f(x) = x$). But even the statement given here is unexpected when first encountered. These two results fit my notion of emergent.

Of course, I've still only hinted at the idea of emergent truth in mathematics. The term "emergent," as I've used it, originated in biology where it means *unexpected, or beyond, reductionist explanation.* Theoretical biology may have a wealth of biochemistry and genetics, but life, what ever that is, remains an emergent phenomena both biologically and mathematically. It is not yet reducible to molecular mechanics. Recent growth in dynamical systems, and the easy computer simulation of complex systems, has lead mathematicians to theoretical biology and biological information processing. These parts of biology are especially rich in ghostly and unexpected phenomena. A search of the world-wide-web using the keywords: *emergent*, *mathematics*, and *biological information processing* returns thousands of hits.

While emergent phenomena are *seen* at the global level, emergent phenomena *arise* from local structure, with no well defined intermediate structure; this is the source of the mystery.

Half a century ago, John von Neumann and Arthur Burks investigated a major biological phenomenon: self-reproduction. Von Neumann's goal was to develop a mathematical insight into the nature of self-reproduction. The formalism he developed evolved into what is we now call cellular automata: networks of small, identical processors carrying out some kind of global computation.

From cellullar automata, we reach possibly the most famous emergent phenomenon: "how does the leopard get its spots?" Allan Turing stated and solved this problem. He wanted to explain how a network of embryonic leopard skin cells can develop a pattern by themselves. This tissue consists of billions of cells, each with the computational complexity of an early personal computer. They exchange information with neighbors, so there is a computer network of sorts. But, being biological in nature, members of this network do not change state in lock-step: the actions of individual cells are not synchronized to those of their neighbors. And there is no regular pattern of communication: one cell might have 16 neighbors while another has 24. Finally, "random cell-activity" means that the computation is non-deterministic!

At the level of cells, or thousands of cells, Turing's network of cells could be said to be amorphous. Yet a color-pattern of similar spots emerges in the fur growing out of the skin. The spots of this pattern contain of tens of millions of cells. And biological evidence indicates that the pattern information telling each cell its place in the pattern does not exist until fairly late in embryonic development. So where does the pattern come from? How does the network compute it?

One might think that there should be a proof that the pattern cannot exist. One would expect that irregularities in coloring should exist on such a small scale that the pelt should be uniformly muddy. Yet using mathematical analysis in a proof similarly to the one that most real numbers are undefinable, one finds the small scale irregularities almost always generating large scale regularities. *While emergent phenomena are* seen *at the global level, emergent phenomena* arise *from local structure, with no well defined intermediate structure; this is the ghostly mathematics.*

Turing first solved the problem in 1952 in a seminal paper now cited as a cornerstone in the development of theoretical biology. It used systems of reaction-diffusion differential equations to explore the mechanisms of morphogenesis. And four decades later, Turing's mechanism was shown to explain pattern generation in vitro.

In 1995, I developed a "measure space" of all legal sequences of global states of cell states for a new solution to Turing's problem. In programming the cells, I used a trick based on an old entropy-reduction idea of Erwin Schrodinger. You may actually watch this mechanism develop a pattern – the simulator is available and explained at <http://www.math.usf.edu/˜stark/>. Being non-deterministic, there are infinitely many possible legal sequences which, with probability 1, eventually produce the leopards' spots. These complex distributed processes work because cells are *not* synchronized, and that makes it model self-organizing

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Leopard Spots

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natural phenomena. Such "asynchronous" activity must be controlled by cell programs which reduce neighborhood entropy (disorder). Entropy-reducing asynchronous networks can do things that synchronized networks cannot! This is certainly a surprising.

When life is viewed as a form of computation and its power is estimated mathematically, it surpass our most powerful supercomputers. In the awesome book "Infinite in All Directions," Freeman Dyson calculates that a 10^{23} bit computation is required for the simplest act by a human (together with the mechanisms of life while performing the act). And a reasonable estimate, by this author in "Computing With Biological Metaphors," of the computational power of our skin is about 1 % or 2 % of that of our brains!

The conceptual distance between physical foundations and these phenomena makes them emergent.

People

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Mourad Ismail has been travelling a lot, including Hong Kong, where he organized a "Summer School in Applied Analysis." He was also appointed an editor of the Journal of Physics A.

Manoug Manougian wrote an award winning documentary "The Genocide Factor: The Human Tragedy." The touching four-part documentary was aired on PBS. He believes that, "If we are going to bring an end to genocide and crimes against humanity, we've got to educate our children."

Yuncheng You published a book on "Dynamics of Evolutionary Equations."

Nagle Lecture Series

F. Alberto Grunbaum, Professor of Mathematics at UC Berkeley, came to USF on Nov. 1 to talk about *Mathematics in medical imaging: the present and the future.* Over 200 people heard his talk about Computerized Tomography (CAT) and Magnetic Resonance Imaging (MRI), and the mathematics behind them. The main problem is how to take the raw data coming from the machine and converting it into an intelligible picture that radiologists can sense of. There are complications, such as: how to make sure that only one picture can come from the data.

Despite the largely medical orientation of the lecture, Professor Grunbaum also discussed security applications, such as airport baggage screening. Professor Grunbaum said that the security technology was several generations behind the medical technology.

Professor Grunbaum is has long studied imaging problems in medical imaging, geophysics, radar detection, and other areas. He has been Chair of the UC Berkeley Mathematics Deptartment and Director of the Center for Pure and Applied Math.

The Nagle Lecture Series was established in honor of the late R. Kent Nagle, a mathematician deeply interested not only in mathematics in itself, but also in mathematics education and the impact of mathe- matics on society. In this spirit, the NLS has invited world renown scholars to speak on such matters in lectures designed for the general public.

Unfortunately, there will be no Nagle Lecture this academic year. The budget is extremely tight, and USF is having difficulty funding even critical operations. The Department will sponsor Nagle Lectures when funding is available. However, the Department does have an account for the Nagle Lecture Series, and we would cheerfully welcome a tax-deductible donation. We would like a reasonable endowment for the program, in order to insulate it from financial bad weather.

Student News

We congratulate the students who received the Bachelor's degree in Mathematics this past year.

In Summer, 2001: Louis Richard Camara, Diane R. Crawford, Cynthia Gomez Martin (Cum Laude), George William Stewart, and John Allen Whitaker (Cum Laude).

In Fall, 2001: Margaret Biebel (Magna Cum Laude), Sarah Elizabeth Emmert, Paul Michael McGill, and Nancy S. McWethy (Magna Cum Laude).

In Spring, 2002: Aaron Anthony Anderson (with Distinction), James P. S. Ascher (Magna Cum Laude), Elizabeth Schofield Dahlquist, LaTisha Candace Jones, Mehul Mehta, Kathleen Elizabeth Mierau, Michelle Madden Turk, and Yousef Yassin Turshani (Magna Cum Laude).

We congratulate the students who received the Master's degree in Mathematics this past year:

Louis Camara, Edgardo Cureg, Stephen Drier, David Kephart, Kalpana Mahalingham, Rodney Taylor, and Kalin Videv.

Let us hear from you

Columbia University Professor (and Nobel Laureate) Isadore Rabi once told Columbia President

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(and future USA President) Dwight Eisenhower that the faculty *are* the university. This is only half true: the students, past, present, and future, are also the university.

We the faculty would like to hear from former students. We maintain a website at

`http://www.math.usf.edu/`

but you can also contact us at

`mathdept@math.usf.edu`

or by phone (813-974-2643) or fax (813-974-2700) or by US mail (Publicity, Department of Mathematics, 4202 E. Fowler Ave. PHY114, Tampa, FL 33620).

A little help from our friends

We are trying to develop a first class teaching and research program with very limited resources. While we are working on regular (official) funding, any little bit we can get helps. If you would like to support the Department, please contact Denise Marks at (813) 974-2643; we can take checks made to the *USF Department of Mathematics*, and if you can designate the funds to go to some place specific (e.g., the Center for Mathematical Services, colloquia, the Nagle Lecture Fund, scholarships, etc.) if you want.