The Quaternion

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The Newsletter of the Department of Mathematics and Statistics

USF UNIVERSITY OF SOUTH FLORIDA

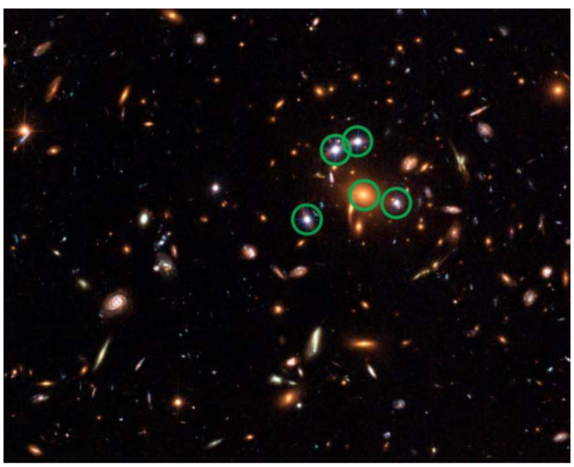
IN THIS ISSUE

The Quaternion is an annual publication of the USF Department of Mathematics and Statistics, which can be visited on the web at <http://www.math.usf.edu/>. Our e-mail address is <mathdept@math.usf.edu>, our snail-mail address is Department of Mathematics and Statistics, University of South Florida, 4202 E. Fowler Ave., PHY114, Tampa, FL 33620. Our phone number is (813) 974-2643, and our fax number is (813) 974-2700.

Gravitational Lensing and the Fundamental Theorem of Algebra

Mathematics is the language of the universe, as Galileo observed, and mathematical revelations often lead to scientific discoveries. One revelation at the University of South Florida led immediately to a discovery lying at the historical heart of relativity theory.

Einstein's general theory of relativity was dramatically confirmed in 1919 when Arthur Eddington observed a total eclipse and reported that the Sun's gravitational field had shifted the images of the stars behind the Sun by the amounts Einstein had predicted. Since then, astronomers have found many "lensed" images of stars, galaxies, and quasars whose radiated light had been slightly redirected by intervening massive objects. Sometimes a lens of massive objects would produce multiple images of a single light source behind them.



A quasar – currently believed to be radiation generated by material as it is consumed by a supermassive black hole in a galaxy ten billion light years away – is lensed by the galactic cluster SDSS J1004+4112, which is seven billion light years away.

We see five images (circled in green) of the quasar as a result of the lensing by the cluster.

Picture produced by the Space Telescope Science Institute, and made available by NASA.

So how many images can a cluster of intervening massive objects produce? If a light source was directly behind a single massive object, the image would be a continuous ring around the massive object. But the number of images produced by a cluster of massive objects appeared to be bounded. In 2001, Sun Hong Rhie predicted that a cluster of $n > 1$ massive objects could generate at most $5n – 5$ images from a single light source.

In 2003, USF Professor Dmitry Khavinson and Professor Genevra Neumann of the University of Northern Iowa, Cedar Falls, posted a paper concerning an upper bound on the number of solutions possible for a certain kind of complex polynomial equation. This was part of a thread of research going back to the Seventeenth century, when mathematicians started asking how many solutions polynomial equations might have. The answer for polynomials of one variable and complex coefficients was provided by C. F. Gauss's Fundamental Theorem of Algebra, but that answer leads to the same question for more complicated classes of equations. And it was an answer for one such class of equations that Khavinson and Neumann provided.

Khavinson and Neumann quickly heard from the astronomers: the solution to the algebraic question confirmed Rhie's prediction that a gravitational lens of $n > 1$ massive objects could generate at most $5n – 5$ images from a single light source.

Mathematics often has eventual applications in science and engineering, but it is gratifying to have an immediate application. For details on lensing and the algebra, see p. 11.

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The Mathematics of Plant Growth

The ultimate form of a plant depends on how the individual cells grew and multiplied during its development, and that development can be modeled by Lindenmayer systems and related modeling techniques. That was the message that Computer Science Professor Przemyslaw Prusinkiewicz of the University of Calgary gave at the R. Kent Nagle Lecture on April 7, 2011.

The models take individual modules – perhaps cells or some other primordial units – that grow and multiply, and models their development. Adjustments of the parameters of their growth will have a dramatic effect on the architecture of the resulting plant. For example, in modeling the growth of a stem, successive leaf nodes can be rotated with respect to their predecessor by an angle, and that angle will affect the ultimate shape of the plant.

Arabidopsis thaliana*, or thale cress, is a widespread annual plant – some dictionaries call it a weed – in the mustard family. Because it reproduces in six weeks and has a simple genome, it is regarded as a "model plant" for studying plant genetics and development. It was an early target of genetic sequencing and was the first higher plant sequenced.*

Professor Prusinkiewicz, with collaborators and students, developed a model of the growth of A. thaliana. The plant is presumed to consist of modules whose growth is modeled by Lindenmayer systems. How the modules grow determines the ultimate shape of the plant.

Professor Prusinkiewicz's primary tool is the Lindenmayer system, introduced by the theoretical biologist Aristid Lindenmayer in the late 1960s. These are built around *production rules*, e.g., for generating a Fibonacci-type structure, one uses the rules A → B and B → AB to get the developing sequence A → B → AB → BAB → ABBAB → BABABBAB → ... (notice that the length of the *n*th term is the *n*th number in the Fibonacci sequence). By using two- and three-dimensional rules, one can obtain models for the growth of two- and three-dimensional structures.

Przemyslaw Prusinkiewicz is a Professor of Computer Science at the University of Calgary, Canada. He is a pioneer of computational modeling, simulation and visualization of plant development, and co-author of *The Algorithmic Beauty of Plants*, which opened this area to a wide audience. His current research is focused on computational models of

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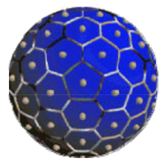
development that link molecular-level processes to the macroscopic form of plants. Professor Prusinkiewicz is a recipient of the Association for Computing Machinery SIGGRAPH *Computer Graphics Achievement Award* and the Canadian *Human Computer Communications Society Achievement Award* for his work pertaining to the modeling and visualization of biological structures. He holds M.Sc. and Ph.D. degrees from the Technical University of Warsaw.

For further information on using Lindenmayer systems to model plant growth, see http://algorithmicbotany.org.

The Nagle Lecture Series was established in honor of the late R. Kent Nagle, a mathematician deeply interested in mathematics in itself, in education and in society. In this spirit, the NLS has invited world renowned scholars to speak on such matters in lectures designed for the general public.



From Dreams to Reality

A Personal Reflection by Manoug Manougian

Before his arrival at the University of South Florida in 1968, Professor Manougian taught at Haigazian College in Lebanon – now Haigazian University – from 1960 to 1962, and later 1964 to 1966. He then returned to the University of Texas at Austin, where he got his Ph.D. He served as chair of the USF Department of Mathematics from 1974 to 1984, and is currently Director of the USF Center for Mathematical Services.

I was in my twenties with a bachelor's degree from the University of Texas at Austin when my fiancée and I headed to Jerusalem to get married. I was offered a teaching position at an Armenian/American institution, Haigazian College, in Beirut, Lebanon. As a math and physics instructor, I was assigned to serve as faculty advisor to the Science Club. Then my childhood dreams came knocking.

It was 1960, and rocketry and space exploration held center-stage in world affairs. The United States and the Soviet Union locked horns for control of space. And I thought, what better way to address current issues and teach the applications and interactions of mathematics and physics than rocketry and space exploration? This prompted me to rename the Science Club as the *Haigazian College Rocket Society* (HCRS).

Freshman and sophomore students joined the club and embarked on a journey of understanding the science of rocketry. The purpose of HCRS was two-fold: to teach students the methods of science through the mathematics and physics of rocketry, and to encourage students to pursue careers in mathematics, engineering and science. The challenge was a driving force!

Lebanon is a small country with majestic natural beauty. One could swim in the Mediterranean in the morning and go skiing among the Cedars of Lebanon in the afternoon. Its capital, Beirut, was known as the Paris of the Middle East. Although the winds of war surrounded Lebanon, the Lebanese were more into commerce, trade and recreational activities.

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I was cognizant of the implications of rocketry, especially in the Middle East. The program was designed specifically as a voyage in the science of rocketry and not as a military venture. No Middle Eastern nation had rockets then: Haigazian College and Lebanon would be the first to launch a rocket in the Middle East.

Soon our experiments led us from primitive one-stage rockets to more sophisticated multistage rockets. By 1962, we developed a viable propellant and perfected separation in flight of multi-stage rockets. Through the President of Lebanon and the Ministry of Education, we received funding and a launching site overlooking the Mediterranean. Our two-stage rockets (Cedar 2 series) had a range of about twelve miles.

By then members of the American University of Beirut joined us, and the Lebanese Rocket Society was born. Between 1963 and 1964, the Society designed, built, and successfully launched two three-stage rockets, the Cedar 3 (below) and Cedar 4. Between

1964 and 1966, the Society designed, built, and successfully launched two three-stage rockets, the Cedar 3 and Cedar 4. Between 1964 and

1964 and 1966, the Society launched powerful one-stage and two-stage rockets; the last of these, the Cedar 8, was a 19-ft two-stage rocket with a range of 90 miles.

Soon, attractive offers intensified to convert the program from a scientific venture into a military one. Being opposed to wars and destruction, for me this was not an option. My family and I returned to Austin, Texas, to complete my graduate studies. Thus ended the Haigazian College Rocket Society program. Some of my students went on to prestigious universities in the U.S. to become successful scientists. As for me, after my graduation in 1968, I chose USF in Tampa, the town that offered Jules Verne his launching site for his novel, *From the Earth to the Moon*.

Now, fifty years later, this part of Lebanon's history has been resurrected. Newspaper and magazine articles appeared from Beirut to Dubai and around the globe. Lebanon issued postage stamps with a replica of the Cedar 4 (below). The London based

New Scientist magazine (March 12 – 18, 2011) carried a full-page article entitled *To Infinity and Beyond,* and lo9.com website carried the headline: *Lebanon was the forgotten player in the sixties space race*, and concluded, "… *the space program remains a remarkable achievement*."

Filmmakers of Abbout Productions are

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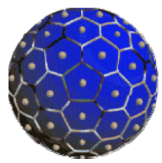
producing a documentary, *From Dreams to Reality*, that tells the story of Lebanon's space program. The documentary is slated to show that the results of scientific experimentation may be used for destructive purposes – or they may be used to enhance our quality of life. Also, at an international art festival in Dubai in March and April of 2011 – with over forty countries participating – an exact replica of our rocket Cedar 4 was displayed at the main entrance: a 22-ft three-stage rocket painted in white to emphasize space exploration and peaceful uses of rockets.

In the words of Albert Einstein, *Imagination is better than knowledge*. That's how creativity is born and innovations created.



Center for Mathematical Services

Every year the USF Center for Mathematical Services offers a six-week summer program for gifted high achieving junior and senior high students. This year, the number of participants increased five-fold over last year, and over 140 people attended the closing ceremony. Financial support came from several sources, including the Philibosian Foundation, The Jagged Peak, Inc. (a former USF student), the Academy of Applied Science, the USF College of Arts & Sciences. The Department of Mathematics & Statistics provided teaching assistants while two publishers, *Cengage Learning* and *Pearson Publishing*, provided the textbooks.

The students were divided into two levels. Level 1 consisted of twenty-three students who are entering grades 9 and 10. They studied biology (taught by Hank Custin), computer science (taught by Andrew Burruss) and mathematics (taught by Professor Manoug Manougian). In the latter course, they were instructed in graph theory, knot theory and, for the first time in the 33-year history of this program, limits, derivatives, and tangent lines were introduced. Level 2 consisted of twenty-six students entering grades 11 and 12. Under dual enrollment, they studied Engineering Calculus 1 (taught by Professor Manougian) and Linear Algebra (taught by Professor Thomas Bieske), as well as biology and computer science.

Four students who are entering grade 12 were selected to work on research projects on a one-on-one basis with a faculty member. The mentors were Professor Mohamed Elhamdadi, Professor Donald Haynie of the USF Physics Department, and Professor Manougian. The four research papers submitted to the Academy of Applied Science are: 1. *Chemical Aspects of Nanomaterials*, by Kyle Griffin; 2. *The Effect of Drag on Artificial Earth Satellites*, by Anthony Ciesla; 3. *Knot Theory, Braids and Cryptography*, by Graham Johnson; and 4. *Theoretical Analysis of Mathematical Models on Electrospinning Mechanic*s, by Xiaodong Lu.

A chess tournament produced a winner in Level 1 and a winner in Level 2, and the 2011 summer program winner was Zhecho Valkanov from Level 1. For the academic year 2011 – 2012, CMS is planning a series of lectures on STEM subjects for high school students in the Tampa Bay Area. Interested high school teachers (and university faculty members) should contact Professor Manougian for further details.

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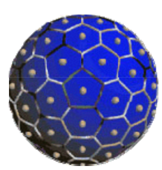
Student News

The following students were awarded Doctoral degrees in June 2010, August 2010, December 2010, or May 2010: Chunling Cong, *Statistical Analysis and Modeling of Breast Cancer and Lung Cancer* under Professor Chris P. Tsokos; Enver Karadayi, *Topics in Random Knots and R-Matrices from Frobenius Algebras* under Professor Masahiko Saito; and Erik Lundberg, *Problems in Classical Potential Theory with Applications to Mathematical Physics* under Professor Dmitry Khavinson.

The following students were awarded Master's degrees in June 2010, August 2010, December 2010, or May 2010: Tharaka Alahakoon; Amber Age, *A Survey of the Development of Daubechies' Scaling Functions* under Catherine Bénéteau; Kenneth Baah; Nana Osei Bonsu; Jonathan Thomas Burns; Andrew Burruss; Won-Seok Choi; Joy M. D'Andrea, *Fundamental Transversals on the Complexes of Polyhedra* under Gregory McColm; Mohammed Anfas LKG Ahamed Hamza; Woo-Suck Jung; Phani Teja Kompella; Jia Liu; Emre Mola; Deniz Ozcan; Albert Paone; Keshav P. Pokhrel; Lauren Polt; Jennifer Tarr, *Domination in Graphs* under Stephen Suen; Katya E. Tipps; Joseph Van Name; Pei-Chen Wu; and Yanling Xi.

The following students were awarded Baccalaureate degrees in June 2010, August 2010, December 2010, or May 2010: Shirley Atori; Mary Ellen Billington, summa cum laude - with honors; Stephanie Chiavaroli - with honors; Gregory Churchill, magna cum laude; Grant Conine - with honors; Megan Corey; Maria Crimi; Jeylynn Cuarezma; Crystal Cullifer; Joshua Davis; Eric Edson; John Felipe; David Flaws, magna cum laude; Jennifer Gilbert; Daniel Gleason; Kathleen Guy, cum laude; Sherry Hayden; Robert Heinsen; Kimberly Kempeneer; Michelle Krause, cum laude - with honors; Donald Lawrence-Hay, magna cum laude; Joshua Lewis; Cody Likavec; Jacob Manning; Janeen Matos; Morgan Mcanally; James O'Connor; Aaron O'Neil, cum laude; Mariam Osman; Narin Persad, cum laude; Katherine Piechocki; Susan Porter; Brittany Poyer; Elena Rodriguez, magna cum laude - with honors; Ryan Rodriguez; Eddie Shivers; Alison Sibol, magna cum laude - with honors; Jamaal Taylor; Benji Torres, cum laude; Christina Trier; Samuel Williams; Nicolle Witte; Trystal Woods; and Besjana Zeqo, cum laude.

We wish them well on their further adventures.



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Math Organization News

The two mathematics organizations at USF are the Florida Epsilon Chapter of the national honor society Pi Mu Epsilon, and the USF chapter of the Mathematical Association of America, the premier mathematics organization for undergraduate mathematics education.

Pi Mu Epsilon

Our Florida Epsilon Chapter of Pi Mu Epsilon inducted eleven new members on April 29 of 2011, most of them mathematics majors or minors. The new inductees (below) were Ryan Arredondo, Shannon Barfield, Hongyi Chen, Hilary English, Kathleen Guy, Donald Hayward, Jing Lin, Ariele Lindemeyer, Andrew Oustimov, Jamie Sprecher, and Timothy Yeatman (at right are Timothy Yeatman, Shannon Barfield, Andrew Oustimov, Jamie Sprecher, and Jing Lin).

The Induction Banquet speaker was Dr. Mike Zaworotko from the USF Dept. of Chemistry, who gave a presentation on "Topology and Crystal Engineering –Where Form Meets Function".

The 2011 PME Outstanding Scholar Awards, which recognize excellence and dedication to mathematics by a graduating senior, were awarded to Pedro E. Gomez, Mary Ellen Billington, and Gregory Churchill (at left). Each of them gave math presentations to

the USF Math Club on topics of their interest: Mary Ellen on "Application of Mathematics to Biology: Using Combinatorial Models to Understand DNA Rearrangements in Cilliates", Greg on "Connecting the Dots: A Game of Graph Theory", and Pedro on "Remarks about Coding a Wavelet-Based Pansharpening Process". Each received a plaque and a $200 scholarship.

The Mathematics & Statistics Department and PME hosted Fall 2010 and Spring 2011 editions of the biannual Hillsborough County Math Bowl. Over four hundred students (and their teachers) from all the high schools in Hillsborough County congregated in the USF Marshall Student Center for a day of friendly (but fierce) team and individual math competitions in the areas of Algebra, Geometry, Pre-Calculus, and Calculus.

In March our Epsilon chapter received a $250 USF Council of Honor Societies grant to buy math books, plaques and trophies to be awarded as PME and USF Math Club prizes.

The 2010-2011 Epsilon Chapter officers were Greg Churchill (President), and Mark Diba (Vice-President). Incoming 2011-2012 President is Ryan Arredondo.

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USF Student Chapter of the Mathematical Association of America

Popularly known as the USF Math Club, this student organization is open for membership to any USF student with an interest in mathematics. Its goals include the diffusion and promotion of mathematics among USF students, mainly undergraduates. 2010-2011 Math Club officers were Tyson Dilorenzo (President), Donald Dahl (Vice-President), and Michael Kummer (Treasurer). Newly elected officers are Jamie Sprecher, Greg Churchill, and Maja Milosevic, in the same rank order. 2010-2011 math club activities included:

Thirteen biweekly meetings during the academic year included free pizza, math announcements and happenings, and math talks by student club members, USF faculty, or invited guests. Highlights included presentations on “What’s at Stake in P versus NP” by Dr. McColm, “Arithmetic Sequences in Sets of Integers” by Dr Totik, “Geometric Approaches to Some Questions of Algebra and Analysis” by Dr. Danielyan, and “From Algebra to Astrophysics” by Dr. Khavinson. Also, “Why did the Mathematician Cross the Road?” by Erik Lundberg, and “Image Processing and Edge Detection” by Robert Veit (both math graduate students), “Cryptography and Knots” by Grant Conine (math undergraduate student), and “Models of Planetary Dynamics for the Hydrogen Atom” by Sean Hollis (engineering undergraduate student).

Eight editions of the popular contest “The Math Problem of the Month” took place last year. Sponsored by the USF math club, this competition is open to all USF undergraduates who want to submit a written solution to a problem posted in the math department hallways at the beginning of each month. The problem is carefully selected by Dr. Milé Krajcevski (one of two faculty math club advisors, the other is Dr. Fernando Burgos) to challenge undergraduate mathematical knowledge and problem-solving skills. Book prizes are awarded to monthly winners, and at the end of the year a “Math Problem Solver of the Year” is selected based on the work during the previous year. This time bragging rights go to Jing Lin, a math and engineering student. Jing received books and a trophy at the PME Induction Banquet in recognition for her year-long math problem-solving achievements.

During the 2011 USF Engineering Expo that took place in February, USF Math Club members manned a booth featuring math exhibits and activities for K-12 students. Activities included i) Babylonian Mathematics (Jamie Sprecher and Tyson Dilorenzo), ii) Learn how People did Math without Computers, and Make your own Math Machine! (Tyson Dilorenzo), and iii) Origami Constructions (Jing Lin). It was a big success and the club is planning to go at it again next year.

As it has become customary, contingents of about eight to ten undergraduate math club members attended the MAA Suncoast Meeting (in December at the University of Tampa), and the MAA Florida Regional Meeting (in February, hosted by Valencia Community College in Orlando) accompanied by faculty math club advisors and long-time math club advisor and founder Dr. Fred Zerla. Math Club and departmental funds supported the trips. Tyson Dilorenzo delivered a student presentation titled “Wavelet Transforms with Lifting” at the Suncoast meeting.

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About Gravitational Lensing and the Fundamental Theorem of Algebra

One of Albert Einstein's primary predictions from his work in "general relativity" (i.e. relativity with forces and hence acceleration) was the deflection of light by gravity. In the simplest case, if a massive object was directly between a star and an observer (Figure 2), light directly from the star would be blocked by the intervening massive object while light from the star at a slight and specific angle would be precisely deflected directly to the observer. Consequently, the observer sees an image of the star deflected from its true position. In Figure 2, since the geometry is symmetric about the axis through the star, the intervening massive object, and the observer, the observer sees deflected images all around the massive object, and hence actually sees a ring about the massive object.

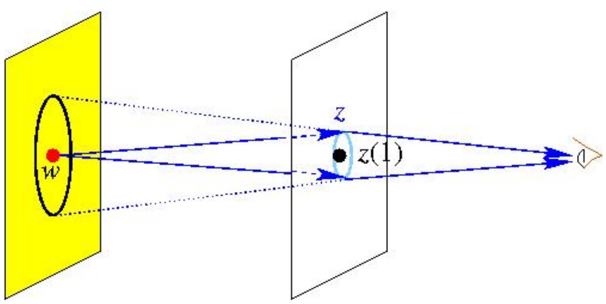

Figure 1. *A massive object (at z(1)) lies directly between a star (at w) and an observer. Light from the star heading just above, below, or towards one or the other side of the observer is bent by the dark object's gravity back towards the observer. The observer sees the light, but perceives the light coming in at various angles: the observer sees a circular halo around the dark object.*

Using Newtonian mechanics, Johann von Soldner predicted in 1804 that light would be bent by gravity, but at the time there was no technology to check his prediction. Regarded as esoteric and relying on unfashionable assumptions, Soldner's work was forgotten. Then in the early Twentieth century, Einstein worked out the deflection angle predicted by his new general theory of relativity and lobbied observatories to check it out. Arthur Eddington's photographs of the 29 May 1919 total eclipse confirmed Einstein's prediction of what we now call *gravitational lensing*, and helped make relativity a household word.

Of course, the star and observer and intervening object rarely line up. If they were not quite aligned, then light could go (almost) directly from the star to the observer, but if the three were almost aligned, there would be a another path that light deflected by the massive object could also take to the observer. Then the observer would see two images of the star, as in Figure 2.

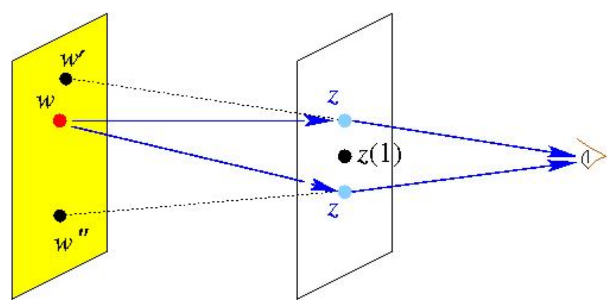

Figure 2. *A dark object lies* almost but not exactly *in between a star and an observer. Light from the star heading* almost *towards the observer is bent by a massive object, and the light that passes through the points z is bent precisely towards the observer so that the observer sees the* two *images at w' and w".*

Space is full of massive objects, some of them dark, and many photographs of distant shining objects show multiple images of those

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those light sources (e.g. the five images of the quasar on page 2). Astronomers would like to use these images to work out the geometry of the intervening massive bodies, so the first question is: how many intervening massive objects might there be?

The problem is that the formula one would have to solve is not nice.

Looking at a special case – all the intervening massive objects are in a relatively compact cluster – one could employ a trick. Imagine that the plane of complex numbers runs right through the cluster, perpendicular to the line of sight from the observer to the star. Project each intervening object to the complex plane, sending a massive object at a point $\mathbf{z}(i)$ to a complex number $z(i)$ as in Figure 3.

Then the formula one would have to solve becomes much nicer.

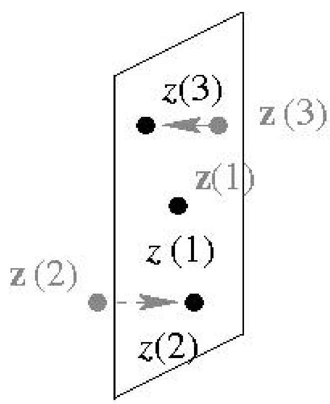

Figure 3. *To model light going through a relatively close crowd of dark objects, we choose a plane perpendicular to the observer's line of sight and project the dark objects at* $\mathbf{z}(1)$, $\mathbf{z}(2)$, *and* $\mathbf{z}(3)$ *onto the plane. We take this plane to be the plane of complex numbers, and hence the projections to be complex numbers* $z(1)$, $z(2)$, *and* $z(3)$. *With this approximation, we proceed with a computation.*

We solve for z in an equation 0 = function(z, $z(1)$, $z(2)$, $z(3)$), *and each solution z represents one location of an image seen by the observer. The simplest question is then: how many solutions z are there? For that will tell us how many images the observer sees.*

In particular, if there is one intervening massive object at $z(1)$, and if we assume that the line of sight from the observer to the star runs through the complex plane at the origin, the problem becomes: solve for the complex numbers z such that

$$0 = z - \sigma/\left(\overline{z} - \overline{z(1)}\right),$$

where σ depends on the mass of the object and the "bar" stands for complex conjugate: $\overline{x + iy} = x - iy$, where $i = \sqrt{-1}$. The first question we can ask is: how many solutions z are there?

More generally, if there are several intervening objects, we get a problem that looks like this: solve for the complex numbers z such that

$$(*) \qquad 0 = z - \overline{p(z)} / \overline{q(z)} \ ,$$

where p and q are polynomials, both of degree at most n (and one of degree n). In 2001, S. H. Rhie predicted that if there were n intervening objects, and $n > 1$, there would be at most $5n - 5$ solutions, i.e. the observer would see at most $5n - 5$ images. Notice that we need $n > 1$, for $n = 1$ can give us a ring of infinitely many images.

In 2003, USF Professor Dmitry Khavinson and Professor Genevra Neumann of the University of Northern Iowa, Cedar Falls, posted a paper proving that for (*), if p and q are relatively prime (no non-constant polynomial divides into both), and if $n > 1$, then the number of solutions is indeed at most $5n - 5$. This confirmed Rhie's prediction. But it turns out that Khavinson and Neumann had been working on something entirely different.

The Fundamental Theorem of Algebra states (using modern terminology) that for any polynomial p with complex coefficients and of degree n, there are n solutions z (counting

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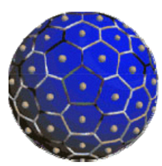
multiplicities) to the equation $p(z) = 0$. The number of roots of an arbitrary polynomial had been a focus of research since the Seventeenth century, and in the end of the Eighteenth century, the Fundamental Theorem was finally proven (although not to our standards) independently by James Wood and Carl Friedrich Gauss. It is Gauss's proof that we remember, and it inspired much work in (a) developing enough mathematics to get a really working proof and (b) counting the number of solutions to other classes of equations. Notice that both endeavors were motivated by purely mathematical concerns.

Following the second endeavor, in the 1990s, T. Sheil-Small and A. S. Wilmshurst raised the question: suppose that r was a "harmonic polynomial"; how many solutions could $r(z) = 0$ have then? This is the sort of question that an explorer might ask when investigating novel terrain: what is going on over here? What about over there? The work of many researchers produces an increasingly fine map of an increasing area of mathematical knowledge.

Anyway, a change of variables allows us to express a harmonic polynomial r in the form $r(z) = p(z) - \overline{q(z)}$, where p and q are polynomials. But such a harmonic polynomial could have many solutions: $r(z) = z - \overline{z}$ has $r(z) = 0$ for all real numbers z. So mathematicians concentrated on special cases, and they got some results, particularly for harmonic polynomials expressible in the form $r(z) = p(z) - \overline{z}$ (Khavinson and Grzegorz Świątek showed that if p was of degree n, then there were at most $3n - 2$ solutions). And from this, as Khavinson and Neumann put it, it is natural to ask the question about functions of the form $r(z) = p(z) / q(z) - \overline{z}$ (so that $\overline{r(z)} = \overline{p(z)} / \overline{q(z)} - z$). And that is how, quite by accident, Khavinson and Neumann solved an open problem in relativity and astronomy.



We'd Like to Hear from YOU!

The Department of Mathematics & Statistics would like to hear from alumni, friends, collaborators, members of the community, and fellow explorers of and guides to the world of mathematics and statistics. Contact us at: 974-2643, or fax 974-2700. E-mail mathdept@math.usf.edu. We have a web-page at http://www.math.usf.edu/. Snail-mail address is Department of Mathematics & Statistics, University of South Florida, 4202 E. Fowler Ave., PHY114, Tampa, FL 33620.

Appeal for funds

We are a growing department in a new university, and we strive to develop new programs to meet the needs and provide opportunities for our students and our community to fulfill their aspirations. With all due respect to Benjamin Franklin, many of the best things in education and scholarship cost money. We would appreciate any assistance we can get from alumni and the community. Feel free to contact our chair, Marcus McWaters, at the above address for details.

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