Consistency and identifiability in stochastic regression models

Christine Jacob
UR341 Mathématiques et Informatique Appliquées, F-78352 Jouy-en-Josas, INRA, France
christine.jacob@jouy.inra.fr

We consider the Conditional Least Squares Estimation (CLSE) of the unknown parameter $\theta \in \mathbb{R}^p$ of a general stochastic regression model $Y_n = f_n(\theta) + \eta_n$, where $Y_n \in \mathbb{R}$, Y_n is $\mathcal{F}_n$-measurable, $f_n(\theta) = E_\theta(Y_n|\mathcal{F}_{n-1})$, and $\mathcal{F}_{n-1}$ is generated by $\{Y_k\}_{k\leq n-1}, \{E_k\}_{k\leq n}$, E_k being an exogenous process such as environment or covariables. This class of models includes many well-known models such as nonlinear regression models, autoregressive processes (ARMA,...), size-dependent branching processes, regenerative processes,....
We assume that $f_n(\theta) = f_n^{(1)}(\mu) + f_n^{(2)}(\mu, \nu)$, where $f_n^{(1)}(\mu) = O(f_n(\theta))$ is the persistent part of the model, as $n \to \infty$, while $f_n^{(2)}(\theta) = o(f_n(\theta))$ is the transient part. We give simple conditions on the model leading to the strong consistency of the CLSE of μ and the weak consistency of the CLSE of ν although $\theta = (\mu, \nu)$ is not asymptotically identifiable. Examples are given in regression and in size-dependent branching processes.

Keywords : Stochastic Regression, Autoregressive Process; Branching Process; Regenerative Process; Conditional Least Squares Estimation

Main Reference:

Jacob C., Lalam N., Yanev N. (2005) Statistical inference for processes depending on environments and application in regenerative processes. *Pliska Stud. Math. Bulgar.*, **17**, 109–136.