The Ramanujan Entire Function

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Abstract

Ramanujan was a selfeducated college drop out who did some of the best mathematics of the twentieth century. He extensively worked on the

$$F(z)=1+\sum_{n=1}^{\infty}\frac{(-z)^n q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)},$$

which we refer to as the Ramanujan entire function. We demonstrate the significance of this function in number theory and analysis and give a new interpretation of the statement

$$1+\sum_{n=1}^{\infty}\frac{z^n q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)}=\prod_{n=1}^{\infty}\left(1+\frac{zq^{2n-1}}{1-c_1q^n-c_2q^{2n}-\dots}\right)$$

in Ramanujan's lost note book.

The coefficients $c_1, c_2, \dots$ turned out to have very interesting paterns and many open problems with be mentioned.